OCR
Oxford Cambridge and RSA

Tuesday 20 June 2017 – Afternoon

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Answer **all** the questions.

1 A curve has equation $y = 2 + e^{\frac{1}{2}x}$. The region R is bounded by the curve and by the straight lines $x = 0$, $x = 4$ and $y = 0$. Find the exact volume of the solid obtained when R is rotated completely about the x-axis. **[5]**

2 **(i)** Use Simpson's rule with four strips to find an approximation to

$$\int_1^9 \ln x \ln(x+4)\,dx,$$

giving your answer correct to 4 significant figures. **[4]**

(ii) Deduce an approximation to

$$\int_1^9 \ln(x^{-1}) \ln(x^2+8x+16)\,dx,$$

giving your answer correct to 4 significant figures. **[2]**

3 **(i)** Sketch the graph of $y = |2x - 7a|$, where a is a positive constant. State the coordinates of the points where the graph meets each axis. **[2]**

(ii) Solve the inequality $|2x - 7a| < 4a$. **[3]**

(iii) Deduce the largest integer N satisfying the inequality $|2\ln N - 10.5| < 6$. **[2]**

4 The angle θ, where $90° < \theta < 180°$, satisfies the equation

$$3\sec^2\theta + 10\tan\theta = 11.$$

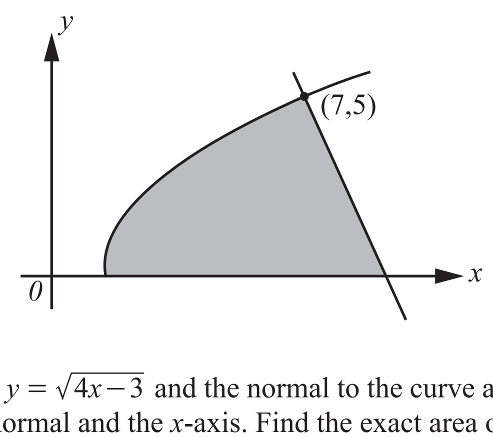
(i) Find the value of $\tan\theta$. **[3]**

(ii) Without using a calculator, determine the value of

(a) $\tan 2\theta$, **[2]**

(b) $\cot(2\theta + 135°)$. **[3]**

5



The diagram shows the curve $y = \sqrt{4x-3}$ and the normal to the curve at the point $(7, 5)$. The shaded region is bounded by the curve, the normal and the x-axis. Find the exact area of the shaded region. **[8]**

6 **(i)** Give full details of a sequence of two transformations needed to transform the graph of $y=\frac{1}{x}$ to the graph of $y=\frac{3}{x+1}$. **[2]**

The function f is defined by $\mathrm{f}(x)=\frac{3}{x+1}$ for $x \geqslant 0$.

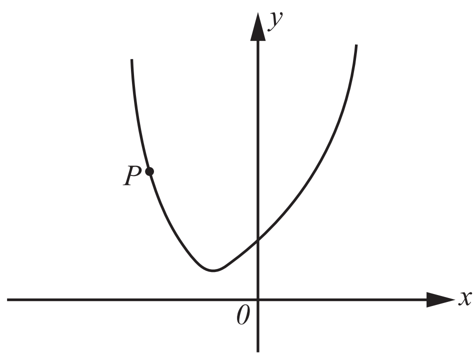
(ii) Determine the range of f. **[2]**

(iii) Find an expression for $\mathrm{f}^{-1}(x)$, and state how the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ are related geometrically. **[3]**

(iv) Solve the equation $\mathrm{ff}(x)=2$. **[3]**

7 **(i)** It is given that $y=a^x$ where a is a positive constant. Express x in terms of $\ln y$ and, by first differentiating x with respect to y, show that $\frac{\mathrm{d}y}{\mathrm{d}x}=a^x \ln a$. **[3]**

(ii)



The diagram shows the curve $y=x^4+4^x$. At the point P on the curve, the gradient of the curve is -8.

(a) Show that the x-coordinate of P satisfies the equation $x=\sqrt[3]{-2-4^{x-1}\ln 4}$. **[3]**

(b) By first using an iterative process based on the equation in part **(a)** with a starting value of -1, find the coordinates of P. Show the result of each step of the iteration process and give the coordinates of P correct to 2 decimal places. **[3]**

8 **(i)** Express

$$3\sin 2\theta \sec\theta + 4\sin 2\theta \operatorname{cosec}\theta$$

in the form $R\sin(\theta+\alpha)$, where $R>0$ and $0^\circ<\alpha<90^\circ$. **[5]**

(ii) Hence solve the equation

$$3\sin(2\beta+20^\circ)\sec(\beta+10^\circ)+4\sin(2\beta+20^\circ)\operatorname{cosec}(\beta+10^\circ)=3$$

for $0^\circ<\beta<360^\circ$. **[5]**

9 **(a)** The equation of a curve has the form $y = \dfrac{px+q}{x^2+3}$. Show that the curve has two distinct stationary points for all non-zero values of the constants p and q. **[4]**

(b) The equation of a curve has the form $y = \mathrm{e}^{x^2}(ax^2+b)$, where a and b are non-zero constants. It is given that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ can be expressed in the form $\mathrm{e}^{x^2}(cx^4+d)$, where c and d are non-zero constants. Prove that $5a+2b=0$. **[5]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance
1			State volume is $\pi\int(4+4e^{\frac{1}{2}x}+e^x)\,dx$	B1	Condone absence of dx; no need for limits here; π may be implied here by its appearance later in solution; integrand must be expanded
			Obtain integral of form $px+qe^{\frac{1}{2}x}+re^x$	*M1	With non-zero constants p, q, r; with or without π here
			Obtain correct $4x+8e^{\frac{1}{2}x}+e^x$ or $\pi(4x+8e^{\frac{1}{2}x}+e^x)$	A1	Or unsimplified equiv; condone presence of + c
			Apply limits 0 and 4 correctly to their integral	M1	Dep *M; with at least one non-zero term obtained from use of limit 0; limits used the wrong way round is M0
			Obtain $\pi(e^4+8e^2+7)$	A1 **[5]**	Or simplified equiv; + c now is A0; ignore subsequent working if necessary
2	i		Attempt calculation of form $k(y_0+4y_1+2y_2+4y_3+y_4)$	M1	Any non-zero constant k with attempts at y values (in terms of ln or decimals); M0 if attempt does not involve exactly four strips; M0 if each y value initially 'amended', to $\ln(2x+4)$ for example
			Obtain $k(\ln1\ln5+4\ln3\ln7+2\ln5\ln9+4\ln7\ln11+\ln9\ln13)$	A1	Or equiv involving decimals indicating use of correct values
			Use $k=\frac{2}{3}$	A1	
			Obtain 26.62	A1 **[4]**	Allow greater accuracy 26.6159…; any value rounding to 26.62 with no errors seen
	ii		State or imply that integrand now involves $-\ln x$ or $2\ln(x+4)$ or both	M1	
			Obtain –53.23 or –53.24 as final answer	A1ft **[2]**	Following their Simpson rule answer from **(i)**, ie -2 times their answer; allow greater accuracy; correct answer with no working earns B2; second use of Simpson's rule leading to correct answer earns B2, but B0 if incorrect; concluding with 53.23 or 53.24 (perhaps with some reference to area below axis) is A0

Question			Answer	Marks	Guidance
3	i		Draw V-shaped graph with vertex on positive x-axis	B1	And graph extending at least a little into second quadrant; condone minimal smoothing at the vertex; allow graph which is asymmetrical about vertical line through vertex unless it is an extreme case
			State $(\frac{7}{2}a, 0)$ and $(0, 7a)$	B1 **[2]**	Can be earned if first B1 not awarded; allow for $\frac{7}{2}a$ and $7a$ marked on axes of graph or cases where zero coordinates are not given but are clearly implied
	ii		Attempt to find two critical values	M1	By squaring both sides (giving 3 terms on left) and solving quadratic equation <u>or</u> by solving two linear equations (one with signs of $2x$ and $4a$ the same and one with the signs different) <u>or</u> using graph with horizontal line representing $y = 4a$
			Obtain $\frac{3}{2}a$ and $\frac{11}{2}a$	A1	
			Conclude with $\frac{3}{2}a < x < \frac{11}{2}a$	A1 **[3]**	Allow the logically correct '$x > \frac{3}{2}a$ and $x < \frac{11}{2}a$' but not conclusions such as '$x > \frac{3}{2}a,\ x < \frac{11}{2}a$'; giving a a particular value means only M1 is available; use of $\le$ signs is final A0
	iii		Relate $\ln N$ to their upper limit of **(ii)** with $a = 1.5$ <u>or</u> proceed directly from inequality in **(iii)** to $2\ln N < 16.5$	M1	
			State the single value 3827	A1 **[2]**	A0 for $N \le 3827$; A0 for $N < 3827.6$
4	i		Use identity $\sec^2\theta = 1 + \tan^2\theta$	B1	Identity must be used not merely quoted
			Attempt solution of 3-term quadratic equation in $\tan\theta$	M1	If using factorisation, M1 earned if their factors correct; if using formula, M1 earned if substitution of their values into correct formula correct; for incorrect equation and two values produced with no working, check that values are correct given their equation so that M1 can be awarded
			Obtain at least $\tan\theta = -4$ from the correct equation	A1 **[3]**	Ignore second value given provided no error at this stage is involved; so $\frac{2}{3}$ and –4 is A1, –4 only is A1, $\frac{2}{3}$ only is A0, $\frac{3}{2}$ and –4 is A0 ; allow solution such as $y = -4$ when clear that y is $\tan\theta$; ignore subsequent work with angles

Question			Answer	Marks	Guidance
	ii	a	Attempt substitution into $\frac{2\tan\theta}{1-\tan^2\theta}$	M1	Using any value from **(i)**
			Use –4 to obtain $\frac{8}{15}$ and no other value	A1	Or exact equiv; full details to be shown; indication of use of calculator is M0; finding $\tan 2\theta$ for both angles is M1A0; answer $\frac{8}{15}$ with no working is M0A0; final answer $\frac{-8}{-15}$ is A0
				[2]	
		b	State or imply $\cot(2\theta+135°)$ is $1\div\tan(2\theta+135°)$	B1	Either at beginning of solution or towards the end
			Attempt substitution of their value from **(a)** into $\frac{1-\tan 2\theta\tan 135°}{\tan 2\theta+\tan 135°}$ or into $\frac{\tan 2\theta+\tan 135°}{1-\tan 2\theta\tan 135°}$	M1	Allow with $\tan 135°$ still present
			Obtain $-\frac{23}{7}$ and no other value	A1	Or exact equiv; full details to be shown; allow $\frac{23}{-7}$
				[3]	
5			Differentiate to obtain $k(4x-3)^{-\frac{1}{2}}$	M1	For any non-zero constant k
			Obtain correct $2(4x-3)^{-\frac{1}{2}}$	A1	Or unsimplified equiv
			Use negative reciprocal of gradient to find intersection of normal with x-axis	M1	Using their attempt at first derivative; <u>either</u> using equation of normal ($y=-\frac{5}{2}x+\frac{45}{2}$) <u>or</u> relevant right-angled triangle
			Obtain $-\frac{5}{2}$ for gradient of normal and hence $x=9$ or equiv such as base of triangle is 2	A1	
			Integrate to obtain $p(4x-3)^{\frac{3}{2}}$	M1	For any non-zero constant p
			Obtain correct $\frac{1}{6}(4x-3)^{\frac{3}{2}}$	A1	Or unsimplified equiv
			Use limits $\frac{3}{4}$ and 7 to obtain $\frac{125}{6}$ for area under curve	A1	Allow calculation apparently using only upper limit
			Use triangle area to obtain $\frac{155}{6}$ for shaded area	A1	
				[8]	

Question			Answer	Marks	Guidance
6	i		Translation parallel to x-axis by – 1	B1	Must use term ‘translate’ or ‘translation’, not ‘move’, not ‘shift’, etc.; translate by $\begin{pmatrix}-1\\0\end{pmatrix}$ is acceptable; ‘in x direction’ is acceptable; ‘translate in negative x direction by –1’ is B0
			Stretch parallel to y-axis, factor 3	B1 **[2]**	Must use term ‘stretch’; ‘in y direction’ is acceptable; condone ‘in y axis’; the two transformations can be given in either order
	ii		State <u>either</u> <3 <u>or</u> >0 or both	M1	Allow any letter; accept < or ≤ , > or ≥ here for method mark
			State correct $0<\text{f}(x)\le 3$ or $0<y\le 3$ or $0<\text{f}\le 3$	A1 **[2]**	
	iii		Obtain expression of form $\frac{a}{x}+b$ or $\frac{a+bx}{x}$	M1	For non-zero constants a and b; or equiv in terms of y
			Obtain correct $\frac{3}{x}-1$ or $\frac{3-x}{x}$	A1	In terms of x now
			Reflection in line $y=x$	B1 **[3]**	Or clear equiv such as one is the mirror image of the other
	iv		<u>Either</u> Attempt correct process to find $\text{ff}(x)$	M1	
			Obtain $\frac{3}{\frac{3}{x+1}+1}$ or $\frac{3x+3}{x+4}$	A1	Or equiv
			Solve to obtain $x=5$	A1	
			<u>Or</u> Attempt $\text{f}^{-1}\text{f}^{-1}(2)$ with their f^{-1}	M1	
			Obtain $\frac{1}{2}$ as first value	A1	
			Obtain 5	A1 **[3]**	

Question			Answer	Marks	Guidance
7	i		State $x = \dfrac{\ln y}{\ln a}$	B1	Ignore any subsequent manipulation of right-hand side
			Differentiate to obtain $\dfrac{dx}{dy} = \dfrac{1}{y \ln a}$	B1	$\dfrac{dx}{dy}$ must be used; quotient rule may be used but must be correct
			Rearrange to confirm $\dfrac{dy}{dx} = a^x \ln a$	B1 **[3]**	AG – at least one intermediate step needed
	ii	a	Obtain derivative $4x^3 + 4^x \ln 4$	B1	Or equiv
			Equate attempt at first derivative to –8 and rearrange to form $x = \sqrt[3]{\ldots}$	M1	Where expression under cube root involves two terms at least one of which involves x; allow M1 if there is one sign slip
			Confirm $x = \sqrt[3]{-2 - 4^{x-1} \ln 4}$	A1 **[3]**	AG – necessary detail needed
		b	Carry out iteration process	M1	Showing at least 3 values after –1
			Obtain –1.27 for x-coordinate	A1	Condone correct value eventually obtained after error in iteration process; answer required to precisely 2 dp; ($-1 \rightarrow -1.277858 \rightarrow -1.272179 \rightarrow -1.272275$); iterates must be present and showing at least 3 dp; answer only and no iterates shown earns 0/3; treat sequence starting at value other than –1 as mis-read
			Obtain 2.79 for y-coordinate	A1 **[3]**	Answer required to precisely 2 dp; using –1.27 to obtain 2.77 is A0; M1A0A1 is possible where iterates shown are not to at least 3 dp(but values are perhaps in calculator)
8	i		Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1	Must be used not merely stated
			Obtain $6\sin\theta + 8\cos\theta$	B1	May be implied
			Obtain $R = 10$	B1	From correct $6\sin\theta + 8\cos\theta$
			Attempt appropriate trigonometry to find α	M1	Allow for $\tan\alpha = \frac{6}{8}$ or equiv
			Obtain 53.1°	A1 **[5]**	Or greater accuracy 53.13…; with no errors seen

Question			Answer	Marks	Guidance
	ii		State or imply equation is $10\sin(\beta+63.1°)=3$	B1ft	Following their R and α
			Carry out correct process to find one value of β	M1	Not available for finding negative angle; must involve use of 2nd quadrant angle
			Obtain $99.4°$ (or $314°$)	A1	Or greater accuracy $99.4122...°$
			Carry out correct process to find second value of β	M1	Must involve use of '5th' quadrant angle
			Obtain $314°$ (or $99.4°$)	A1	Accept value rounding to 314 providing no error; and no others between 0 and 360
				[5]	[Note: Solving $10\sin(\theta+53.1°)=3$ can earn M1 M1 if correct processes followed; if continue to find correct angles by subtracting $10°$, A1 A1 available; B1 can be retrospectively given even if answers are wrong]
9	a		Differentiate using quotient rule or equiv	M1	With negative sign in numerator, with $(x^2+3)^2$ in denominator and at least one of the two terms in the numerator correct
			Obtain $\dfrac{p(x^2+3)-2x(px+q)}{(x^2+3)^2}$ or equiv	A1	
			Equate derivative to zero and attempt discriminant	M1	Provided equation is a 3-term quadratic with p and q present
			Obtain $4q^2+12p^2$ and observe it is positive	A1 **[4]**	With at least one reference to squared value being positive
	b		Differentiate to obtain form $e^{x^2}(px^3+qx)$	M1	
			Obtain $\dfrac{dy}{dx}=2xe^{x^2}(ax^2+b)+2axe^{x^2}$	A1	Or equiv
			Obtain $\dfrac{d^2y}{dx^2}=e^{x^2}(4ax^4+10ax^2+4bx^2+2a+2b)$	A1	Or equiv
			Equate coefficient of $x^2e^{x^2}$ to zero	M1	Provided second derivative involves $e^{x^2}x^4$, $e^{x^2}x^2$ and e^{x^2} terms and no others
			Confirm $5a+2b=0$	A1 [	AG – necessary detail needed